Analytic regularization of the Yukawa Model at Finite Temperature

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Abstract

We analyse the one-loop fermionic contribution for the scalar effective potential in the temperature dependent Yukawa model. In order to regularize the model a mix between dimensional and analytic regularization procedures is used. We find a general expression for the fermionic contribution in arbitrary spacetime dimension. It is found that in $D = 3$ this contribution is

finite.

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1　Introduction

Recently there has been much interest in the phase structure of theories involving scalar fields presenting spontaneous symmetry breaking. Many aplications have been done in the Weinberg-Salam model and in grand-unified theories. The temperature generally is the parameter whose variation induces the transition from the broken to the unbroken phase, at least for the most current systems that develop first or second order phase transitions.

To describe a second order phase transition the variation of the mass with the temperature is the most important fact. On the other hand the dependence of the coupling constant with the temperature may induce a first order phase transition in the scalar sector, as suggested by two of the authors in a recent work [1].

We start from the Yukawa model and we analyse the contribution coming from the fermionic loops for the temperature dependent scalar effective potential. The ultraviolet divergences are dealed with the method of analytic regularization [2]. We recall that the basic idea of this technique is to replace the denominator of the propagator $(p^2 - m^2 + i\epsilon)$ by $(p^2 - m^2 + i\epsilon)^{1+\alpha}$ where α is the regulating parameter initially taken to be large enough. Consequently in a open connected set of points in the complex plane α the Feynman amplitudes are analytic. Then it is possible to analytically continue the Feynman expressions to the whole complex plane. In the Laurent expansions of these expressions we can identify the counterterms as the polar terms in the analytic extensions at some points of the complex plane.

To deal with finite temperature field theory using the imaginary time formalism we will have to use dimendional regularization in the momenta and deal with the Matsubara sums using another method. The most popular method to deal with the Matsubara sum is an analytic extension away from the discrete complex energies down to the real axis, with the replacement of the energies sums by countour integrals [3]. If we are interested in systems at high temperature the decoupling theorem [4] allow us to use the dimension reduction method (DR). This approach has been used by many authors [5]. The basic idea is that in the imaginary time formalism the free propagator has a form $(\omega_n^2 + p^2 + m^2)^{-1}$. The Matsubara frequency act like a mass so in the high temperature regime the non-static $(n \neq 0)$ modes decouple, and we have a three dimensional theory. In other words, the only modes whose contribution do not fall of exponentially at distances much greater than β are the $(n = 0)$ modes of the bosons. Integration over the fermionic modes and the non-zero modes of the bosons result in a three dimensional theory. Of course this effective model will describe the original model only for distances $R >> \beta$. As was stressed by Landsman [6] the standard summation method [3] based on analytic continuation do not work in the dimensional reduction approach. Instead, we have first to compute momentum integrals using dimensional regularization and to deal with the Matsubara sums, a inhomogeneous zeta function analytic regularization has to be performed.

Recently such technique has been used to study different models at finite temperature. Ford and Svaiter [7] and Malbouisson and Svaiter [1] [8] studied the $\lambda\varphi^4$ and the Efimov-Fradkin (truncated or not) model at finite temperature. The possibility of vanishing the temperature dependent

coupling constants in these models has been investigated. In the first paper, assuming a non-simply connected spatial section, the thermal and topological contributions to the renormalized mass and coupling constant in the $(\lambda\varphi^4)_{D=4}$ model was obtained at the one-loop approximation. In the second one, the authors extend the discussion of the massive self-interacting $\lambda\varphi^4$ model to an arbitrary D-dimensional spacetime with trivial topology of the spacelike sections. The main result is that the possibility of a first order phase transition drived by the temperature dependent coupling constant, in the region where the model is super-renormalizable arises. The discussion in the case of a scalar model with non-polynomial interaction Lagrange density (the Efimov-Fradkin model) has been done in the third work. For $D > 2$ it was proved that at least two coupling constants of the truncated model may vanish and become negative by effect of temperature changes, while in the non-truncated model all the coupling constants remain positive for any temperature. The method used in the above quoted papers could provide an almost natural way to investigate stability regimes in finite temperature QFT models.

It has been often suggested that the thermal contributions to the renormalized coupling constants of quantum models may bring up non-trivial effects. For instance, Gross, Pisarski and Yaffe [9] argue that in finite temperature $(QCD)_4$ the effective coupling constant $g(\Lambda)$ decreases as the temperature or density is raised. In fact, they show in a perturbative context that at the first non-trivial order $(QCD)_4$ should be asymptotically free at high temperature or pressure. In this approximation it is expected that at high temperatures thermal excitations produce a plasma of quarks and gluons which screen all (color) eletric flux. Such a transition from a low temper-

ature confined phase to a high temperature color screening phase has been also investigated by Polyakov [10] and Susskind [11] and others in lattice gauge theories. Such results have important astrophysical applications in the study of neutron stars or primeval universe models.

The main goal of this paper is to investigate the one-loop femionic contribution to the scalar effective potential at finite temperature assuming that bosons and fermions interact via a Yukawa coupling. The outline of the paper is the following: in section II we briefly review the formalism of the effective potential. In section III the fermionic contribution to the effective potential is obtained. In section IV the singularity structure of the one-loop fermionic contribution to the scalar effective potential is studied Conclusions are given in section V. In this paper we use $\hbar = k_B = c = 1$.

2 The effective action and the effective potential at zero temperature.

In this section we will briefly review the basic features of the effective potential associated with a real massive self-interacting scalar field at zero temperature. Although the formalism of this section may be found in standard texbooks, we recall here its main results for completeness. Let us consider a real massive scalar field $\varphi(x)$ with the usual $\lambda \varphi^4(x)$ self-interaction, defined in a static spacetime. Since the manifold is static, there is a global timelike Killing vector field orthogonal to the spacelike sections. Due to this fact, energy and thermal equilibrium have a precise meaning.

For the sake of simplicity, let us suppose that the manifold is flat. In the path integral approach, the basic object is the generating functional,

$$Z[J] = < 0, out|0, in > =$$

$$\int \mathcal{D}[\varphi] \exp\{i[S[\varphi] + \int d^4x J(x)\varphi(x)]\} \tag{1}$$

where $\mathcal{D}[\varphi]$ is an apropriate integration measure and $S[\varphi]$ is the classical action associated with the scalar field. The quantity $Z[J]$ gives the transition amplitude from the initial vacuum $|0, in >$ to the final vacuum $|0, out >$ in the presence of some source $J(x)$, which is zero outside some interval $[-T, T]$ and inside this interval is switched adiabatically on and off. Since we are interested in the connected part of the time ordered products of the fields, we take the connected generating functional $W[J]$, as usual. This quantity is defined in terms of the vacuum persistent amplitude by

$$e^{iW[J]} = Z[J], \tag{2}$$

and the connected n-point functions $G_c^{(n)}(x_1, x_2, .., x_n)$ are

$$G_c^{(n)}(x_1, x_2, .., x_n) = \frac{\delta^n W[J]}{\delta J(x_1)...\delta J(x_n)}|_{J=0}. \tag{3}$$

Expanding $W[J]$ in a functional Taylor series, the n-order coefficient of this series will be the sum of all connected Feynman diagrams with n external legs, i.e. the connected Green's functions

6

defined by eq.(3). Then

$$W[J] = \sum_{n=0}^{\infty} \frac{1}{n!} \int d^4x_1..d^4x_n \ G_c^{(n)}(x_1, x_2....x_n)J(x_1)J(x_2)..J(x_n). \tag{4}$$

The classical field $\varphi_0(x)$ is given by the normalized vacuum expectation value of the field

$$\varphi_0(x) = \frac{\delta W}{\delta J(x)} = \frac{< 0, out|\varphi(x)|0, in >_J}{< 0, out|0, in >_J}, \tag{5}$$

and the effective action $\Gamma[\varphi_0]$ is obtained by performing a functional Legendre transformation

$$\Gamma[\varphi_0] = W[J] - \int d^4x J(x)\varphi_0(x). \tag{6}$$

Using the functional chain rule and the definition of φ_0 given by eq.(5) we have

$$\frac{\delta \Gamma[\varphi_0]}{\delta \varphi_0} = -J(x). \tag{7}$$

Just as $W[J]$ generates the connected Green's functions by means of a functional Taylor expansion, the effective action can be represented as a functional power series around the value $\varphi_0 = 0$, where the coeficients are just the proper n-point functions $\Gamma^{(n)}(x_1, x_2, .., x_n)$ i.e.,

$$\Gamma[\varphi_0] = \sum_{n=0}^{\infty} \frac{1}{n!} \int d^4x_1 d^4x_2...d^4x_n \ \Gamma^{(n)}(x_1, x_2, .., x_n) \ \varphi_0(x_1)\varphi_0(x_2)..\varphi_0(x_n). \tag{8}$$

The coefficients of the above functional expansion are expressed in terms of the connected one-particle irreducible diagrams (1PI). Actually, $\Gamma^{(n)}(x_1, x_2, ..., x_n)$ is the sum of all 1PI Feynman diagrams with n external legs. Writing the effective action in powers of momentum (around the point where all external momenta vanish) we have

$$\Gamma[\varphi_0] = \int d^4x \left(-V(\varphi_0) + \frac{1}{2}(\partial_\mu \varphi)^2 Z[\varphi_0] + .. \right). \tag{9}$$

7

The term $V(\varphi_0)$ is called the effective potential [12] .To express $V(\varphi_0)$ in terms of the $1PI$ Green's functions, we write $\Gamma^{(n)}(x_1, x_2, .., x_n)$ in momentum space,

$$\Gamma^{(n)}(x_1, x_2, .., x_n) = \frac{1}{(2\pi)^n} \int d^4k_1 d^4k_2 .. d^4k_n (2\pi)^4 \delta(k_1 + k_2 + .. k_n) \ e^{i(k_1 x_1 + .. k_n x_n)} \tilde{\Gamma}^{(n)}(x_1, x_2, .., x_n).$$

$$(10)$$

Assuming that the model is translationally invariant, i.e. φ_0 is constant over the manifold, we have

$$\Gamma[\varphi_0] = \int d^4x \sum_{n=1}^{\infty} \frac{1}{n!} \Big(\tilde{\Gamma}^{(n)}(0, 0, ...)(\varphi_0)^n + ... \Big).$$

$$(11)$$

If we compare eq.(9) with eq.(11) we obtain

$$V(\varphi_0) = - \sum_n \frac{1}{n!} \tilde{\Gamma}^{(n)}(0, 0, ..)(\varphi_0)^n,$$

$$(12)$$

then $\frac{d^n V}{d\varphi_0^n}$ is the sum of the all $1PI$ diagrams carring zero external momenta. Assuming that the fields are in equilibrium with a thermal reservoir at temperature β^{-1}, in the Euclidean time formalism, the effective potential $V(\beta, \varphi_0)$ can be identified with the free energy density and can be calculated by imposing periodic (antiperiodic) boundary conditions on the bosonic (fermionic) fields.

3   The one-loop effective potential of the Yukawa model at zero and finite temperature.

Let us consider a system consisting of bosons and fermions fields interacting via a Yukawa coupling in thermal equilibrium with a reservoir at temperature β^{-1}. They are defined on a four dimensional flat spacetime with trivial topology of the spacelike sections. In the zero temperature case the generating functional for the scalar and fermionic fields correlation functions is given by:

$$Z[\eta, \bar{\eta}, J] = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}\varphi \exp\{i[S[\bar{\psi}, \psi, \varphi] + \int d^4x \bar{\psi}\eta + \bar{\eta}\psi + J\varphi]\} \tag{13}$$

where $\bar{\psi}(x)$, $\psi(x)$, $\bar{\eta}(x)$ and $\eta(x)$ are elements of the Grassmann algebra and $\varphi(x)$ and $J(x)$ are commuting variables.

The perturbativelly renormalizable action has the form,

$$S[\bar{\psi}, \psi, \varphi] = \int d^4x \left(\frac{1}{2}(\partial_\mu \varphi_b)^2 - \frac{1}{2}m_0^2\varphi_b^2 + V(\varphi_b) + \bar{\psi}_b(i \not{D} - M_0 - g_0\varphi_b)\psi_b\right) \tag{14}$$

where $V(\varphi_b) = \frac{\lambda}{4!}\varphi_b^4$ and m_0, M_0 are respectivelly the boson and the fermion bare masses and λ_0 and g_0 are the bare coupling constants. Of course φ_b and ψ_b are bare bosonic and fermionic field.

The most general divergent terms are of the type [13]

$$-\Gamma^{div} = \int d^4x \left(\frac{1}{2}\delta Z_\varphi (\partial_\mu \varphi)^2 - \frac{1}{2}\delta m^2 \varphi^2 + \right.$$
$$\left. +\delta Z_\psi i\bar{\psi}(i \not{D}\psi - \delta M\bar{\psi}\psi - g\delta Z_g \bar{\psi}\psi\varphi + \frac{1}{4}\delta\lambda\varphi^4 + \frac{1}{3}\delta\sigma\varphi^3 + \delta c\varphi\right) \tag{15}$$

Although the action is renormalizable, the model is not multiplicatively renormalizable. To circumvect this difficulty and to allow the theory to become multiplicatively renormalizable we shall introduce at the tree level action all terms which we expect to be generated by the renormalization

procedure, i.e.

$$S(\bar{\psi}, \psi, \varphi) = \int d^4x \left(\frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - \sum_{n=1}^{4} \lambda_n \varphi^n + \bar{\psi}(i \not{\partial} - M - g\varphi)\psi \right) + counterterms, \qquad (16)$$

where $\lambda_2 = \frac{1}{2}m^2$, $\lambda_3 = \frac{\sigma}{3!}$ and $\lambda_4 = \frac{\lambda}{4!}$.

As usual, perturbation theory is generated by,

$$Z(\bar{\eta}, \eta, J) = exp\{-i \int d^4x (i^3 g \frac{\delta^3}{\delta\eta\delta\bar{\eta}\delta J} + V(\frac{\delta}{\delta J}))\} Z_0(\bar{\eta}, \eta, J) \qquad (17)$$

where

$$Z_0(\bar{\eta}, \eta, J) = exp-i \int d^4x d^4y (\bar{\eta}(x)\Delta_F(x - y)\eta(y) + \frac{1}{2}J(x)\Delta(x - y)J(y)) \qquad (18)$$

with $\Delta_F(x - y)$ and $\Delta(x - y)$ being respectively the fermionic and bosonic propagator functions,

$$\Delta_F(x - y) = (i \not{\partial}_x + M)\Delta(x - y), \qquad (19)$$

and

$$\Delta(x - y) = \frac{1}{(2\pi)^4} \int d^4p \frac{e^{-ip(x-y)}}{p^2 - m^2 + i\epsilon}. \qquad (20)$$

From the above formulas, following a porcedure entirely analogous to that described in the preceding section for the pure scalar case it is easy to get the fermionic contribution to the effective potential $V(\varphi_0)$

10

$$V(\varphi_0) \int d^4x = i \ln \int_0^0 d\bar{\psi} d\psi \ exp\{i \int d^4x \bar{\psi}(i\ \not{\partial} - M - g\varphi)\psi\} \tag{21}$$

After a Wick rotation to Euclidean space and using the rules for Grassmann integrals we get the contribution from the single fermionic loops to the scalar effective potential

$$V(\varphi_0) \int d^4x = -\ln \det(i\ \not{\partial}_E - M - g\varphi_0) \tag{22}$$

Using a well known result

$$\log \det(M + g\varphi_0) = tr\ \log(M + g\varphi_0), \tag{23}$$

we have,

$$\ln \det(i\ \not{\partial}_E - M - g\varphi_0) = tr\ \log(i\ \not{\partial}_E) - \sum_{s=1}^{\infty} \frac{(-i)^s}{s}(M + g\varphi_0)^s tr(\frac{1}{\not{\partial}_E})^s. \tag{24}$$

Using a Fourier representation for $\frac{1}{\not{\partial}_E}$ and taking into account that the contributions from odd values of s in the above sum vanish, it is possible to recast the fermionic contribution to the effective potential in the form [14],

$$V(\varphi_0) = 4 \sum_{s=1}^{\infty} \int \frac{d^4p}{(2\pi)^4} \frac{(-1)^s}{2s} \frac{(M + g\varphi_0)^{2s}}{(p_E^2)^s}. \tag{25}$$

In the finite temperature case using the Matsubara formalism we have to perform the replacements $\omega \to \omega_n = \frac{2\pi}{\beta}(n + \frac{1}{2})$ and $\frac{1}{2\pi}\int dq_E^0 = \frac{1}{\beta}\sum_n$. Then the contribution from the single fermionic

loops to the effective potential is given by

$$V(\varphi_0, \beta) = \frac{2}{\beta} \sum_{s=1}^{\infty} \sum_{n=-\infty}^{+\infty} \int \frac{d^d p}{(2\pi)^d} \frac{(-1)^s}{s} \frac{(M + g\varphi_0)^{2s}}{(\omega_n^2 + q^2)^s}. \tag{26}$$

Let us define the quantities,

$$a = (\frac{1}{\beta\mu})^2 \tag{27}$$

$$\phi = \frac{\varphi_0}{2\pi\mu} \tag{28}$$

and

$$\gamma = \frac{M}{2\pi\mu}, \tag{29}$$

where μ is a parameter with mass dimension introduced to deal with dimensionless quantities performing analytic extensions. First we use dimensional regularization going to a generic D-dimensional spacetime. Then eq.(26) becomes

$$V(\phi, \beta) = \mu^D \sum_{s=1}^{\infty} a^{\frac{D}{2}-s} f(D, s)(\gamma + g\phi)^{2s} \sum_{n=-\infty}^{\infty} \frac{1}{((n+\frac{1}{2})^2)^{s-\frac{d}{2}}}. \tag{30}$$

where $f(D, s)$ is given by:

$$f(D, s) = \frac{2\pi^{\frac{d}{2}}}{\Gamma(s)} \Gamma(s - \frac{d}{2}) \frac{(-1)^s}{(2\pi)^{2s}}. \tag{31}$$

12

Before going one some commments are in order. It is well known [15] that dimensional regularization techniques for massless fields can not led to definite results due to the presence of infrared divergences [15]. Since we are regularizing only a $d = D - 1$ dimensional integral, this procedure is equivalent to inserting a mass into the d dimensional integral. In other words, the Matsubara frequencies play the role of "masses" in the integral provided that we exclude the limit $\beta \to \infty$ which means that we *must restrict ourselves to non-zero temperatures*. Another point is that in order to evaluate the one-loop finite temperature diagrams the usual approach is to express the integrand as a countour integral [3]. In this paper we use another technique still aplying the principle of the analytic extension.

In the next section we will analyse the singularity structure of the inhomogeneous Riemann zeta function and other factors appearing in eq.(30) in order to identify the divergent terms in the fermionic contribution to the effective potential. We start by analytically regularizing the model.

4   The singularity structure of the fermionic contribution to the effective potential.

As we remarked before the fermionic contribution to the effective potential is ill defined due to the singularities of the gamma function that appears in $f(D,s)$ and the singularities in the Matsubara sum. The Matsubara sum may be expressed in terms of the generalized inhomogeneous

Riemann zeta function, which can be analytically extended to a meromorphic function in the whole complex s plane. The polar terms must be removed in the renormalization procedure. In order to identify these poles let us first recall the definition of the inhomogeneous Riemann zeta function or Hurwitz zeta function [16]

$$\zeta(z, q) = \sum_{n=0}^{\infty} \frac{1}{(n+q)^z}, \tag{32}$$

which is analytic for $Re(z) > 1$.

After some manipulations it is possible to express the Matsubara sum in eq.(30) in terms of $\zeta(z, q)$ and write $V(\phi, \beta)$ in the form,

$$V(\phi, \beta) = 2\mu^D \sum_{s=1}^{\infty} a^{\frac{D}{2}-s} f(D, s)(\gamma + g\phi)^{2s} \zeta(2s - d, \frac{1}{2}). \tag{33}$$

To analytically extend the inhomogeneous Riemann zeta function, we go along the following steps: first using the Euler representation for the Gamma function we write it as

$$\zeta(z, q) = \frac{1}{\Gamma(z)} \int_0^{\infty} dt \ t^{z-1} \frac{e^{\frac{-t}{2}}}{1 - e^{-t}}. \tag{34}$$

Next, we split the integral from zero to infinity in two integrals, from zero to one and from one to infinity. The second one is an analytic function of z, the divergences being associated to the zero limit of the first integral. Then using a Bernoulli representation for the integrand it is possible to get the following expression to the analytic extension of $\zeta(z, \frac{1}{2})$

14

$$\zeta(z, \frac{1}{2}) = g_1(z) + \frac{1}{\Gamma(z)} \sum_{n=0}^{\infty} \frac{B_n(\frac{1}{2})}{n!} \frac{1}{z+n-1} \tag{35}$$

where $g_1(z)$ is given by

$$g_1(z) = \frac{1}{\Gamma(z)} \int_1^{\infty} dt \, t^{z-1} \frac{e^{\frac{t}{2}}}{e^t - 1}, \tag{36}$$

and the $B_n(x)$ are the Bernoulli coefficients [16]. We remark that in the literature there is another formula for the analytic extension of the inhomogeneous Riemann zeta function; the Hermite formula [17] given by

$$\zeta(z, q) = \frac{1}{2q^z} + \frac{q^{1-z}}{z-1} + 2 \int_0^{\infty} (q^2 + y^2)^{\frac{-z}{2}} \sin(z \arctan \frac{y}{q}) \frac{1}{e^{2\pi y} - 1} dy. \tag{37}$$

Of course the analytic extension must be uniquely defined and these are only different representation of the same analytic extension. Substituting the analytic extension given by eq.(35) in the fermionic contribution to the effective potential $V(\phi, \beta)$ we get,

$$\begin{aligned}
V(\phi, \beta) &= \mu^D \sum_{s=1}^{\infty} a^{\frac{D}{2}-s} h(D, s)(\gamma + g\phi)^{2s} \frac{1}{\Gamma(-\frac{D}{2}+s+1)} \\
&\quad \left(\int_1^{\infty} dt \, t^{2s-D} \frac{e^{\frac{t}{2}}}{e^t-1} + \sum_{n=0}^{\infty} \frac{B_n(\frac{1}{2})}{n!} \frac{1}{2s-D+n} \right).
\end{aligned} \tag{38}$$

where the regular function $h(D, s)$ is given by,

$$h(D, s) = 2 \frac{(-1)^s}{s} \frac{(2\pi^{\frac{1}{2}})^{D-4s}}{\Gamma(s)}. \tag{39}$$

Let us analyse the two cases $D = 3$ and $D = 4$ separately. For the case $D = 3$ we have

$$V(\phi, \beta) = \mu^3 \sum_{s=1}^{\infty} a^{\frac{3}{2}-s} h(3,s)(\gamma + g\phi)^{2s} \frac{2}{\Gamma(-\frac{3}{2}+s+1)}$$
$$\left(\int_1^{\infty} dt \, t^{2s-3} \frac{e^{\frac{t}{2}}}{e^t - 1} + \sum_{n=0}^{\infty} \frac{B_n(\frac{1}{2})}{n!} \frac{1}{2s-3+n} \right)$$

(40)

The fermionic contibution to the effective potential is finite. There is no ultraviolet divergences

in $D = 3$. One would not normally expect this since the tadpole graph is ultraviolet divergent

$(s = 1)$. This situation is very similar to the calculation of the renormalized vacuum energy

of scalar fields confined in boxes (Casimir energy) [18]. Dolan and Nash used the zeta function

analytic regularization method to obtain the Casimir energy of conformally coupled scalar field

confined in odd and even dimensional spheres [19]. They obtained that for odd dimensional

spheres (even space-time dimension) there is a pole in the point of interested, being necessary the

introduction of a counterterm, while for even dimensional spheres (odd dimensional space-time)

the result obtained is naturally finite. No renormalization is needed. For a careful study of this

subject see ref.[20].

For the case $D = 4$ we have

$$V(\phi, \beta) \;=\; \mu^4 \sum_{s=1}^{\infty} a^{2-s} h(4,s)(\gamma + g\phi)^{2s} \frac{1}{\Gamma(s-1)}$$
$$\left(\int_1^{\infty} dt \, t^{2s-4} \frac{e^{\frac{t}{2}}}{e^t - 1} + \frac{B_0(\frac{1}{2})}{2s-4} + \frac{B_2(\frac{1}{2})}{4s-4} + \sum_{n=3}^{\infty} \frac{B_n(\frac{1}{2})}{n!} \frac{1}{2s-4+n} \right).$$

(41)

Note that the factor $\Gamma^{-1}(s - 1)$ just cancels the pole from the term $n = 2$ in the sum over n.

The pole comming from the term $n = 0$ in the sum must be canceled by the introduction of a

suitable counterterm. All other terms $s \geq 3$ are finite.

5 Conclusion

The aim of this paper is to discuss an alternative method to deal with the Matsubara sum in a finite temperature field theory with bosons and fermions in interaction. We use this method to calculate the one-loop fermionic contribution to the scalar effective potential assuming the Yukawa coupling between fermions and bosons. Note that we are using a BPHZ scheme with subtraction at zero momentum of the Feynman integrals. Matsumoto, Ojima and Umezawa [21] claims that tha Matsubara method seems to produce temperature dependent divergences which disapear only after a summation over the Matsubara sums. We showed that the countertems are temperature independents.

A curious observation is in order. We note that eq.(38) does not contains singularities for *any* odd space time dimension D, due to the fact that the sum is over integer values of s, and the Bernouilli coefficients $B_n(\frac{1}{2}) = 0$ for n odd. For even values of D the fermionic contribution to the effective potential (see eq.(38)) has only a divergence due to the term $s = \frac{D}{2}$, $n = 0$.

It would be interesting to generalize the method if we consider that there is a non-zero fermion density [22]. This can be done introduzing a chemical potential σ. At finite temperature the chemical potential will change the Matsubara frequencies by $\omega_n \rightarrow \omega_n + i\sigma$ [23]. In this case we have to analytically extend the inhomogeneous Epstein zeta function $\zeta(z, q)$ for complex q. This subject is under invetigation.

6 Acknowlegement

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